

# Möbius Strip Experiments

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You will need:

1. Lots of strips of paper approximately 12" long and about 2 to 3" wide
2. Glue
3. Scissors
4. Pen or pencil
5. Patience and willingness to experiment

One auxiliary skill that can be imparted is the value of taking notes with each experiment.

## Experiment #1:

Take a strip and twist it once, joining the two ends. Starting at any point in the middle of the width of the strip, take a pen (or pencil) and trace a line along the length without lifting the pen. You may need to keep the pen on the surface and "feed" the paper under it. Notice that eventually the line finishes at the starting point without ever crossing the edge of the strip. This shows that the strip has only one "side."

## Experiment #2:

This is the same as E1, but instead of twisting the strip once, twist it twice before gluing the two ends together. Once again, trace a line down the middle of the strip; what happens now? How many "sides" does the strip have now?

## Experiment #3:

Same as E2, but this time twist the strip three times before gluing. When you trace the line down the middle, how many sides are there now?

## Question #1:

Does the number of sides depend on the number of twists? If one twist and three twists are "odd," and two twists are "even," does the number of sides depend on whether there are an even or odd number of twists?

#### Experiment #4:

Go back to the single twist strip from E1. Take your scissors and carefully cut along the line in the middle of strip. What happens? How many Möbius strips do you have now? Are they free of each other, or entangled? Write down your answer.

#### Experiment #5:

Go back to the double twist strip from E2. Once again, take your scissors and carefully cut along the line in the middle of the strip. What happens now? Is the result the same or different from E4?

#### Experiment #6:

Using the triple twist strip from E3, once again cut along the line. Is the result the same or different from E4 and E5?

**Question #2:** Is there a pattern that depends on the number of twists, odd or even?

#### Experiment #7:

Start with a fresh strip with only one twist before gluing. Now start a line about  $1/3$  from one edge, and continue it all around the strip until it connects. Then start a new line about  $1/3$  from the other edge, and do the same thing. You should now have two parallel lines which never intersect. With your scissors, now cut along each line individually. What is the result? Describe carefully.

#### Experiment #8:

Repeat E7, but this time use a double twist band as in E2. What happens this time?

#### Experiment #9:

Repeat E7, but this time use a triple twist band as in E3. Are the results the same or different?

**Question #3:**

Are the patterns that you observe for two cuts different from the patterns you observe for only one cut?

**Question #4:**

What would you predict would happen if you took a longer and wider strip—say 16” long and 4” wide—and did the one, two, and three twist experiments with three lines each 1” apart, cutting each of the original strips 3 times along the lines?

It is very interesting to try to make the prediction first, and then to do the experiment to see if your prediction turns out to be correct. You can view the first nine experiments as “data gathering” to try to see patterns, and then to formulate a “theory” about what happens when you go to a situation that you have not seen before. Experiments 10, 11, and 12 then will allow you verify if your predictions based on the theory are true or not.

This is exactly the way scientists work. They gather data, try to see patterns, formulate a theory, and then do an experiment to either validate or invalidate the theory. They build from simple cases to more complicated ones. One feature is indelibly illustrated: if the experiment fails, then the theory is wrong. One experiment is worth a thousand opinions, and we learn something whether the experiment succeeds or fails.

About 40 years ago, a middle-school student did this set of experiments for a Science Fair project when he was in the seventh grade. He mounted his results on a poster-sized board, explaining at each step what he had done. His dad’s only contribution was to propose the experiments; the youngster figured out the patterns on his own. I believe the project was well received, because it demonstrated a lot of science with a minimum of materials and at an extremely low cost.