## Tangramion ${ }^{\text {T" }}$

The happy fusion of Tangrams and Stomachion


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## A brief history



The Tangramion puzzle is the creation of Serhiy Grabarchuk, the Ukrainian puzzle creator and chronicler whose whole family of talented puzzlists has added immensely to the puzzle lore of our world. Their various books and websites are listed on the References page.

Serhiy evolved the Tangramion set through a detailed analysis of three classic puzzles-Tangrams (above left), Archimedes' Square (above center, also known as Stomachion) and the T. A. Snider Diamond Puzzle (shown on page 5).

Tangrams are believed to have originated in China centuries ago. They are popular all over the world and are used in many schools to excite kids about math. The seven tiles are famous for making countless other shapes, though the square has essentially only one solution.

Archimedes' Square has an amazing, more than 22-century history and is, in fact, the oldest documented puzzle in the world. See the References for the thrilling story of the decipherment of an ancient manuscript, the Archimedes Palimpsest, in the first decade of the $21^{\text {st }}$ century. Its Greek name, Stomachion, hints at how difficult the ancient players found it to be, that it gave people a stomach ache. It was not until 2003 that Bill Cutler, a puzzle designer himself, proved that Stomachion has 536 solutions.

The Archimedes Palimpsest's original texts hint at two different versions of Archimedes' Stomachion: square and rectangular. Since the latter is just a square version stretched to a $1 \times 2$ rectangle, and squares are more traditional, Serhiy used the square in his research.

Archimedes' puzzle contains 14 tiles, including two duplicate pairs. Their shapes are quite awkward and inconvenient, with unusual angles and lengths of sides that are complicated to match up, especially since no piece is symmetrical. Archimedes' Stomachion is thus more of a mathematical curiosity than a really fun puzzle.

One day, comparing Archimedes' Square (diagram a) and the T. A. Snider Diamond Puzzle (b), Serhiy discovered that surprisingly many of their dissection lines matched exactly, and several tiles were exactly the same. Moreover, if in the traditional drawing of Archimedes' Square we exchange two blocks of tiles (c), we can easily spot even more elements the puzzles share. This is surprising, since both of them use a very unusual underlying grid, where no piece is symmetrical. In particular, the


Diamond Puzzle has its diagonals running at 45 and 26.565... degrees, something almost never used in other Tangram-like puzzles. And the very same diagonals occur in Archimedes' Square.

In the third diagram (c), the main dividing lines (heavy lines) isolate six clearly visible regions
 exactly coinciding with regions of the Diamond Puzzle and containing from one to four original tiles of Archimedes' Stomachion.

Now we can see how to fuse some of the original tiles and reduce the number of pieces, making them easier to use, and thus the puzzle is more playable
 and fun. Further, with 14 tiles in Archimedes' Square and 7 pieces in Tangrams, having just 7 tiles in the new puzzle was very appealing.

And so, adding a short vertical line to the whole overlaid pattern (c), Serhiy finally achieved his goal of 7 tiles. Lastly, combining the two puzzle names, Stomachion and Tangram, gave us the name for this new puzzle, Tangramion. The square has exactly 10 solutions, verified by computer. Its main difficulty is that no piece is symmetrical, even though they look close. Watch carefully to place only exact same-length sides together. Often you may need to flip them over to fit.

## Puzzle figures

In addition to the square, many other figures can be made, using all 7 tiles. How many of these can you solve? We thank Serhiy Grabarchuk, Tanya Grabarchuk and Helen Grabarchuk for contributing most of these designs.

How many new designs can you discover? Send us your most interesting ones, and we'll add them to the collection with your name.

## Convex polygons

Some 93 convex polygons have been identified and solved to date. Can you find others? We've arranged them here by their number of sides.

3 sides


4 sides


6

5 sides


## 6 Sides



## 7 sides



8 sides


Note that there are very shallow angles in the way these solutions fit together, giving the figures their eight facets.

The figures are all drawn to scale, and all are different. Some may look very close, but there are subtle differences in their lengths and angles. For example, the two identical triangles of the set are not isosceles. The lengths of their two longest sides differ ever so slightly, and flipping them will give a different figure. Your best clue is that the precisely cut pieces must match exactly edge to edge, corner to corner.

Bonus challenge: Is it possible to create a convex Tangramion figure with more than 8 sides?

## Symmetrical shapes

Mirror symmetry. Symmetrical Tangramion shapes are fairly rare. To date, we've found 33 figures with mirror symmetry - the 30 below and the square plus 2 others among the convex shapes (pages 6-9). Can you find them?


Rotational symmetry. In addition to the 20 figures below, find the 11 rotationally symmetrical shapes among the convex figures on pages 6-8, and 5 others among the letters on page 12. Rotational symmetry happens when you turn a figure halfway around a center point, like a clock face, and it makes the same shape again. The letters $S$ and $Z$ are examples of rotational symmetry.


Pseudo-symmetry. The figures at the top of page 12 look almost symmetrical, but they're really leaning to one side, or are a bit shorter on one edge, or have slightly different angles.


## A partial alphabet...



A nice challenge: Solve these, then construct some of the missing letters. Send us your designs and they'll appear on websites across the globe. Email a digital photo to: kadon@gamepuzzles.com, with the subject line, "Tangramion letters".

## Digits and things

12345以58才

## Boats and planes

Let your artistry take flight...


13

Birds and beasts
Let your imagination soar! Create your own zoo...


## Explorations

- We don't yet know how many different convex polygons (no indented angles) are possible with the 7 Tangramion tiles. We show 93, including the square, in this book. Send us any new ones and we'll send you a little prize.
- How about purely symmetrical figures? We show 68 besides the square. How many new ones can you find? Look for mirror or rotational symmetry. The square is the only figure which has both.


## References

To read more about this exciting historical project and Serhiy's other puzzle creations, check out these references:

- www.archimedes-lab.org/latin.html\#archimede
- www.archimedespalimpsest.org/about/
- www.ams.org/ams/netz-jmm2003.html
- www.ageofpuzzles.com/Puzzles/Tangramion/Tangramion.htm
- www.grabarchukpuzzles.com/
- www.peterpuzzle.com/ | www.unipuzzle.com/


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Serhiy Grabarchuk's Age of Puzzles: Puzzle Fields; pilot edition, privately published by Serhiy Grabarchuk Puzzles, 2011.

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